HYDRAULIC TURBOMACHINES
Exercises 1 - Hydraulic Energy

1.1 Specific energy loss calculations

Kaplan turbine of Ligga III power station in Sweden could be mentioned as featuring one of the highest capacities for a Kaplan turbine, 182 MW. The layout of the power plant is shown in Figure 1. Technical data are given in Table 1. For the calculation, use the following values as the gravity acceleration and water density:

$$g = 9.81\ \mathrm{m \cdot s^{-2}},\ \rho = 1000\ \mathrm{kg \cdot m^{-3}}$$

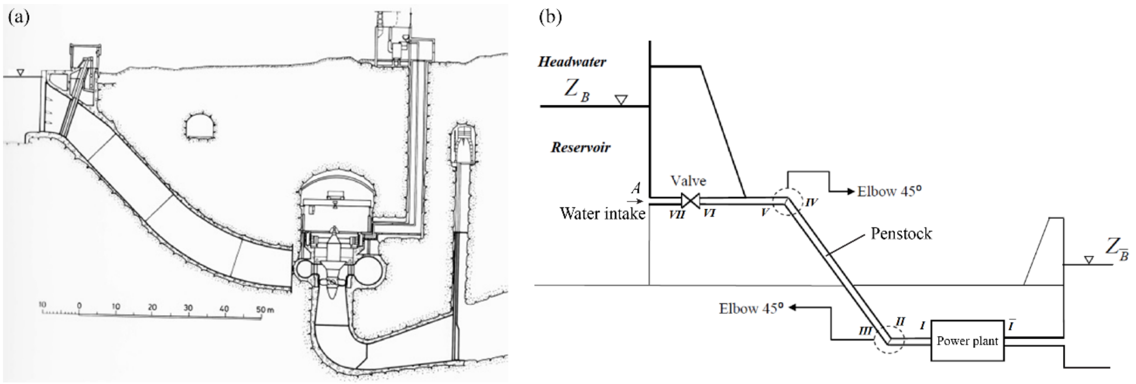

Figure 1 – (a) Meridional view of the Ligga III power plant and (b) simplified layout of the power plant for specific energy loss calculations

Table 1 Technical data

Data	Symbol	Value	unit
Headwater reservoir level	Z_B	122	(m)
Tailwater reservoir level	$Z_{\bar{B}}$	73	(m)
Water kinematic viscosity	ν_w	10^{-6}	($m^2\ s^{-1}$)
Rated discharge	Q	516	($m^3\ s^{-1}$)
Penstock length	L_p	156.1	(m)
Penstock diameter	D_p	7.5	(m)
Roughness	k_s	$45 \cdot 10^{-6}$	(m)
Intake loss coefficient*	$k_{r,intake}$	1.0	(-)
Elbow loss coefficient*	$k_{r,elbow}$	0.15	(-)
Valve loss coefficient*	$k_{r,valve}$	0.10	(-)
Number of poles	z_p	72	(-)
Grid frequency	f_{grid}	50	(Hz)
Output Torque	T	20.54	(MNm)

* *With respect to the specific kinetic energy of the penstock*

1) Calculate the potential specific energy $gH_B - gH_{\bar{B}}$ assuming that the atmospheric pressure is constant.

$$gH_B - gH_{\bar{B}} = \left(\frac{p_B}{\rho} + gZ_B + \frac{C_B^2}{2}\right) - \left(\frac{p_{\bar{B}}}{\rho} + gZ_{\bar{B}} + \frac{C_{\bar{B}}^2}{2}\right)$$

$$= \left(\cancel{\frac{p_{atm}}{\rho}} + gZ_B + \varepsilon^2\right) - \left(\cancel{\frac{p_{atm}}{\rho}} + gZ_{\bar{B}} + \varepsilon^2\right)$$

$$= g\left(Z_B - Z_{\bar{B}}\right) = 480.69 \ \mathrm{J \cdot kg^{-1}}$$

Here ε represents a very small value, as the velocity in the reservoir can be considered infinitely small.

2) By using the Churchill formula and the energy loss coefficients given in *Table 1*, calculate the energy losses of the installation $\sum gH_r$ for the rated discharge. The specific energy losses $gH_{r,\bar{I} \div \bar{B}}$ in the tail race channel, between $\bar{I}$ and $\bar{B}$ can be neglected.

The specific energy budget yields:

$$\sum gH_r = \sum gH_{r,B \div I} + \underbrace{\sum gH_{r,\bar{B} \div \bar{I}}}_{\approx 0}$$

$$= \left(k_{r,intake} + k_{r,valve} + k_{r,elbow} + k_{r,elbow} + \lambda \frac{L_p}{D_p}\right) \cdot \frac{C^2}{2}$$

It is necessary to compute the local loss coefficient with Churchill formula for the rated discharge.

$$C = \frac{Q}{A_p} = \frac{4Q}{\pi D_p^{\ 2}} = 11.68 \ \mathrm{m \cdot s^{-1}}$$

$$Re = \frac{CD_p}{\nu_w} = 8.76 \cdot 10^7$$

$$A = \left[2.457 \cdot \ln \frac{1}{\left(\frac{7}{Re}\right)^{0.9} + 0.27 \frac{k_s}{D_p}}\right]^{16} = 1.3394 \cdot 10^{24}$$

$$B = \left[\frac{37530}{Re}\right]^{16} = 1.2885 \cdot 10^{-54}$$

Which yields $\lambda = 7.7 \cdot 10^{-3}$ and the energy losses:

$$\sum gH_r = 106.43 \ \mathrm{J \cdot kg^{-1}}$$

3) Calculate the turbine specific energy E, the net available head H and the hydraulic power P_h for the rated discharge.

$$E = gH_I - gH_{\bar{I}} = g\left(Z_B - Z_{\bar{B}}\right) - \sum gH_r$$

$$= 374.23 \ \mathrm{J \cdot kg^{-1}}$$

$$E = gH_1 - gH_{\bar{1}} = gH$$

$$H = \frac{E}{g}$$
$$= 38.15 \text{ m}$$

$$P_h = \rho Q E$$
$$= 193.10 \text{ MW}$$

4) Calculate the rotating frequency of the runner n.

$$n = \frac{2 f_{grid}}{z_p} \cong 1.39 \text{ Hz}$$

5) Calculate the machine power output P and the global efficiency η.

$$P = \omega \cdot T = 2\pi n \cdot T = 2\pi \cdot 1.39 \cdot 20.54 = 179.39 \text{ MW}$$

$$\eta = \frac{P}{P_h} = \frac{179.39}{193.10} = 0.929$$

The operating condition of the power plant is modified, with a new discharge value $Q_{new} = 398 \text{ m}^3 \cdot \text{s}^{-1}$ and a new elevation of the headwater reservoir $Z_{B\_new} = 135 \text{ m}$.

6) Assuming that the specific energy losses of the installation are proportional to the square of the discharge, calculate the new specific energy losses induced by the change of the operating condition.

$$\frac{\sum gH_r^{new}}{\sum gH_r^{old}} = \left[\frac{Q^{new}}{Q^{old}}\right]^2 \; yielding \; \sum gH_r^{new} = \sum gH_r^{old} \left[\frac{Q^{new}}{Q^{old}}\right]^2 = 63.32 \text{ J} \cdot \text{kg}^{-1}$$

7) For this operating condition, the turbine output power is found to be $P = 210 \text{ MW}$, compute the new global efficiency.

$$P_h = \rho Q^{new} E^{new} = \rho Q^{new}\left(gZ_B^{new} - gZ_{\bar{B}} - \sum gH_r^{new}\right) = 216.87 \; MW$$

$$\eta = \frac{P}{P_h} = 96.83 \text{ \%}$$

2 TRANSFORMED SPECIFIC ENERGY

Here, the fundamentals of hydraulic power plants and the calculation of the transformed specific energy E_t are studied. The general sketch of a hydraulic power plant with a pump-turbine unit is shown in Figure 1. The pump-turbine is operated in turbine mode at the best efficiency point. The points 1 and $\bar{1}$ correspond to the inlet and the outlet of the turbine, respectively. For gravity acceleration and density, use the following values:

$g = 9.81 \text{ m s}^{-2}$, $\rho = 1'000 \text{ kg m}^{-3}$

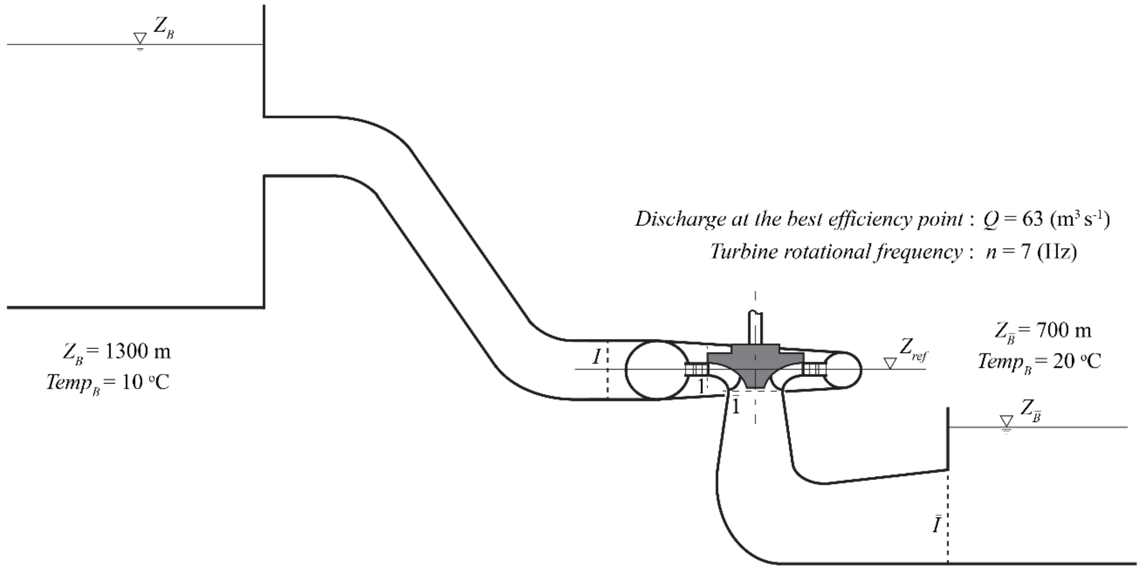

Figure 2 - Entire installation of a pump-turbine

1) Assuming that the atmospheric pressure p_a is constant, express the potential specific energy $E_{potential}$ by g, Z_B and $Z_{\bar{B}}$. Then, calculate the value.

$$E_{potential} = g\left(Z_B - Z_{\bar{B}}\right) = 5886\,\mathrm{J\,kg^{-1}}$$

2) For a practical study, the atmospheric pressure changes depending on the altitude and temperature. Considering the change of the atmospheric pressure, express the potential specific energy $E_{potential}$ by g, ρ, Z_B, $Z_{\bar{B}}$, $p_{a,B}$ and $p_{a,\bar{B}}$. Then, calculate the value of $E_{potential}$.

   It should be noted that the atmospheric pressure at an altitude h (m) and temperature T (°C) can be calculated by the following equation:

$$p_a = p_0\left(1 - \frac{0.0065h}{T_0 + 273.15}\right)^{5.257}$$

$$p_0 = 101.3\,\mathrm{kPa},\quad T_0 = T + 0.0065h$$

$$p_{a,B} = 86.791\ kPa$$

$$p_{a,\bar{B}} = 93.421\ kPa$$

$$E_{potential} = \left\{\left(gZ_B + \frac{p_{a,B}}{\rho}\right) - \left(gZ_{\bar{B}} + \frac{p_{a,\bar{B}}}{\rho}\right)\right\} = 5879.37\ \mathrm{J\,kg^{-1}}$$

3) Express the available specific energy E using necessary variables among $E_{potential}$, $gH_{rB\div I}$, $gH_{rI\div 1}$, $gH_{r\bar{1}\div\bar{I}}$, and $gH_{r\bar{I}\div\bar{B}}$.

$$E = E_{potential} - gHr_{B\div I} - gHr_{\bar{I}\div\bar{B}}$$

4) Express the transformed specific energy E_t using any necessary variables among $E_{potential}$, $gH_{rB\div I}$, $gH_{rI\div 1}$, $gH_{r\bar{1}\div\bar{I}}$, and $gH_{r\bar{I}\div\bar{B}}$.

$$E_t = E_{potential} - gHr_{B\div I} - gHr_{\bar{I}\div\bar{B}} - gHr_{I\div 1} - gHr_{\bar{1}\div\bar{I}}$$

5) The transformed power P_t is defined by $P_t = \rho Q_t E_t$. Q_t is the discharge passing through the turbine, and it is lower than the discharge Q. Describe the reason of this.

The discharge leaks through the clearance between the rotational and stationary parts (turbine and casing), therefore the discharge passing through the turbine Q_t is lower than the discharge Q.

6) The transformed power P_t can be written as a function of the available power P: $P_t = \frac{1}{\eta_{me}} P$ with η_{me} the mechanical efficiency defined by $\eta_{me} = \eta_m \cdot \eta_{rm}$, where η_m is the efficiency of the bearing and η_{rm} the efficiency of the disc friction. Express the transformed power P_t as a function of mechanical efficiency η_{me}, global efficiency η, density ρ, discharge Q and available energy E.

$$P_t = \frac{1}{\eta_{me}} P = \frac{1}{\eta_{me}} \eta P_h = \frac{\eta}{\eta_{me}} \rho Q E$$

7) Introducing the volumetric efficiency and the energetic efficiency defined as $\eta_q = \frac{Q_t}{Q}$ and $\eta_e = \frac{E_t}{E}$, respectively, express the global efficiency η as a function of η_e, η_q, η_m, and η_{rm}.

$$\eta = \eta_m \eta_{rm} \eta_e \eta_q$$

8) Assuming that the losses $gH_{rB \div I} + gH_{r\bar{I} \div \bar{B}}$ correspond to 5% of the potential specific energy, calculate the hydraulic power P_h.

$$P_h = \rho Q E = \rho Q \cdot 0.95 \cdot E_{potential} = 351.88 \text{ MW}$$